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Effective Length Factor for Column in Frame with Girders on Elastic Foundation

F.Y. Al-Ghalibi

Abstract— This paper considered the effects of elastic foundation on effective length factor calculations using subassembly model for braced and unbraced frames, the girders far ends condition are modeled as rigid, fixed, or hinged. The derivation is under same assumptions of conventional effective length factor (K-factor) using two approaches. The first approach considered the effects of elastic foundation and girders far ends conditions by depending on β1 and β2 parameters, the general modified K-factor equations have been derived for braced and unbraced frame using slop-deflection method. In the second approach, formulae of stiffness modification parameter 'γ', which modify the relative columns to beams stiffness factor 'G' are calculated with direct use of G in the US National codes alignment chart. Approximate modified K-factor formulae are proposed for braced and unbraced frames. The Approximate proposed modified K-factor equations are suitable for practical use. Numerical examples are presented to illustrate the effects of elastic foundation taken into consideration girders far ends conditions. The Results showed that the amount of elastic foundation modulus influenced the modified K-factor values and the stiffness of elastic foundation must be taken into account in K-factor calculations. Generally, modified K-factor values decrease with increasing of stiffness of the elastic foundation.

Index Terms— Stability, stiffness, elastic foundation, modified K-factor, stability functions, braced-frame, unbraced frame.

1 INTRODUCTION

THE importance of K-factor calculation has increased since the middle of the last century. K-factor formulations and applications are widely used in U.S. specifications such as AISC-LRFD, AISC-ASD, ACI (318), AASHTO and many textbooks. In the structural engineering, the calculation process of effective length factors are one of the most important applications spatially in the field of second order analysis and members slenderness. K-factor is widely used in the field of advanced structural analysis, design, buckling issues and structural stability. The studies, research and many references showed K-factor importance and effects on the behavior of structural analysis and design. The value of K-factor is variable and it's depending upon many factors, those factors are related to structure's dimensions, types of supports, frame's geometry, shape of members, type of member's material, and loading's case. The derivation of K-factor equation is based on calculation assumptions, these assumptions are simplified the modeling and calculation of K-factor. For example, the assumption of simultaneously buckling of all columns in one story with an idealized subassembly model used to prepare the alignment charts, this assumption was proposed by Julian and Lawrence(1959). LeMessurier(1977)modified Julian and Lawrence's assumption based on considering the columns in a one story buckle simultaneously and the strong column braced the weak column or that carry high axial load. Depending on this assumption, LeMessurier applied some types of correction factor to the alignment charts.

• *University Assistance Lecturer, Structures and Water Resources Department, College of Engineering, Kufa University, AN-Najaf city, AN- Najaf province, Iraq. Tel: +964 781 9877878, E-mail: furatyh@yahoo.com.*

Lui(1992)proposed simple and more effective method in evaluating the K-factor in sway frame, the method considered both instability of member and frame. Hu and Lai (1986) proposed a computer programming method to calculate the K-factors. In that computer program, the modeling of element was considered the effects of axial force for typical offshore structures with ends rotational spring and translational elastic springs at a distance from element end. Yura(1971)studied the K-factor calculation for unbraced frame, his strategy involved reducing the calculation process by modeling the problem as an equivalent pinned-ends braced columns. Duan and Chen(1988)proposed a modification to K-factor's calculation for braced framed column to increase the accuracy of K-factor's calculation by presenting a modification to G factors which are used in US National codes alignment charts by considering the effect of columns far-ends condition in the above and below the column under consideration. Also, Duan and Chen (1988) modified K-factor calculation for unbraced framed column by taking into account the effect of columns far ends condition in the above and below the considered column. Their work involved modifying G factors used in US National codes alignment charts. Chen et al(1993a) suggested a new method to calculate K-factor for braced and unbraced column in frame restrained by tapered girder with various girders far end conditions. The model improved the conventional G factor by girder stiffness modification parameter α. Chen et al (1993a) discussed the ACI (318-89) simplified equations and indicated some comments and limitations used for simplified ACI-code K-factor equations. Dumonteil (1992)discussed the exact conventional K-factor formulae for braced and unbraced columns and check the accuracy of K-factor approximate equations. Dumonteil (1999)discussed some historical K-factor equations for braced and unbraced columns. He checked the accuracy with the exact form for the French "CM 66" approximate formulae for braced and un-

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braced columns, Donnell's approximate formulae and Newmark's approximate formulae for braced column.

The analysis of beams on elastic foundation was well established in the literature. The differential equation approach used by Hetenyi (1946) is practical for engineering purposes. In structural buildings, some elements may be interacting with elastic foundation; such interaction affected the accuracy of K-factor calculation. This paper presents the exact and approximate modified K-factor equations for braced and unbraced column in frame with girders on elastic foundation. The girders far ends are modeled as rigid, fixed, and hinged. The modified K-factor exact formulae derived by using two approaches, the first approach considered the parameter βn which is derived using subassembly model (Fig.1). The value of parameter βn is varying with frame's case (braced or unbraced), elastic foundation stiffness parameter λ, and girders far ends condition. The second approach considered the conventional alignment charts for prismatic girders, so that the original G factor can't use directly. The present study developed G factor used in US codes alignment charts by depending on stiffness parameter γn which is derived for girders on elastic foundation with various far end conditions by dividing the bending stiffness of girder on the elastic foundation on the bending stiffness of ordinary member. The proposed modification to G factor allow to use the US cods alignment charts for column in frame with girders on elastic foundation and various far end conditions. The exact modified K-factor formulae are derived according to the following assumptions:

1. Columns buckle simultaneously.
2. All members are elastic and prismatic cross section.
3. All girders have negligible axial force.
4. All column ends are rigid, while the girders far end are modeled as rigid, fixed or hinged.
5. All columns have equal stiffness parameter $L\sqrt{P/EI}$.
6. For braced frame, angles of rotation at opposite girder ends have an equal value and produce single bending curvature as shown in Fig.1(B); whereas for unbraced frame, angle of rotation at opposite girder ends also have equal value and but produce reverse bending curvature as shown in Fig.1(C).
7. Distribution of resistance of joint is proportion to I/L of two columns in above and below the joint.

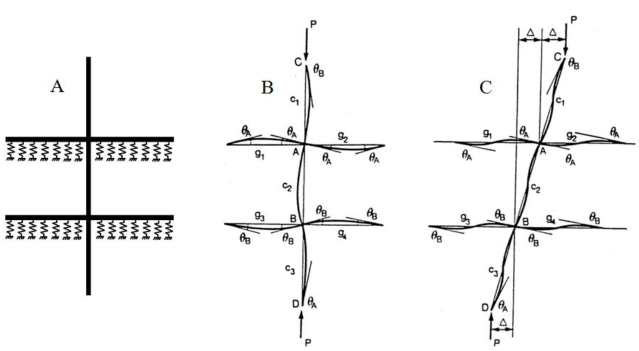

Fig.1.Subassembly model for a frame with girders on elastic foundation, (A) modified model, (B) modified braced frame subassembly model, (C) modified unbraced frame subassembly model

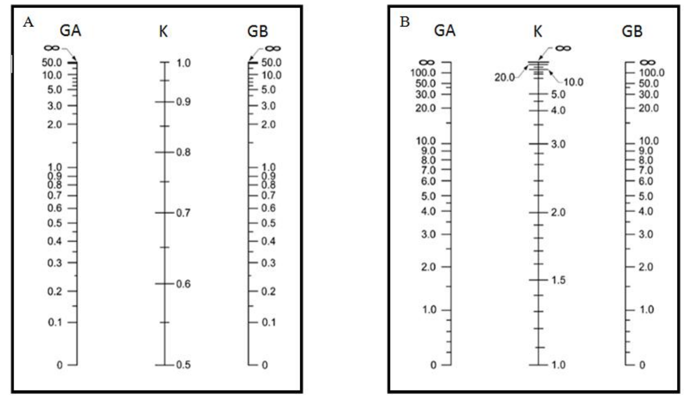

Fig. 2. Alignment charts of the US National cods, (A) braced frame, (B) unbraced frame

2 Exact modified K-factor equations (exact first approach)

The differential equation of girder on elastic foundation can be given as:

$$EI\frac{d^4y}{dx^4}+k_s y=0 \tag{1}$$

From the solution of Eq.1, the stiffness coefficients of girder on elastic foundation ($\overline{S}$ & $\overline{Sc}$) can be obtained as follow:

$$\overline{S}=2\lambda\frac{Sinh(\lambda)\ Cosh(\lambda)-Sin(\lambda)\ Cos(\lambda)}{Sinh^2(\lambda)-Sin^2(\lambda)} \tag{2}$$

$$\overline{Sc}=2\lambda\frac{Sin(\lambda)\ Cosh(\lambda)-Sinh(\lambda)\ Cos(\lambda)}{Sinh^2(\lambda)-Sin^2(\lambda)} \tag{3}$$

Where parameter λ is given by:

$$\lambda=\sqrt[4]{\frac{k_s}{4EI}} \tag{4}$$

Where, k_s is an elastic foundation parameter

2.1 Braced frame

At the buckling load, the columns have an axial force parameter

$$\rho=\left(\frac{1}{K}\right)^2$$

and the stability functions S_K and SC_K stated elsewhere(Duan and Chen, 1988, Duan and Chen, 1989) as shown in Eq.5 and Eq.6:

$$S_K=\frac{\left(\frac{\pi}{K}\right)Sin\left(\frac{\pi}{K}\right)-\left(\frac{\pi}{K}\right)^2Cos\left(\frac{\pi}{K}\right)}{2-2Cos\left(\frac{\pi}{K}\right)-\left(\frac{\pi}{K}\right)Sin\left(\frac{\pi}{K}\right)} \tag{5}$$

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$$SC_K = \frac{\left(\frac{\pi}{\overline{K}}\right)^2 - \left(\frac{\pi}{\overline{K}}\right)Sin\left(\frac{\pi}{\overline{K}}\right)}{2-2Cos\left(\frac{\pi}{\overline{K}}\right) - \left(\frac{\pi}{\overline{K}}\right)Sin\left(\frac{\pi}{\overline{K}}\right)} \tag{6}$$

By using the slop-deflection equations for the subassembly model for column in braced frame on elastic foundation with variable girders far end conditions, the following relation can be achieved:

$$\begin{vmatrix} GBSC_K & \beta_2 + GBS_K \\ \beta_1 + GAS_K & GASC_K \end{vmatrix} = 0 \tag{7}$$

Eq. 7 can be written as:

$$\frac{\beta_1\ \beta_2}{GA\ GB} + S_K\left(\frac{\beta_1}{GA} + \frac{\beta_2}{GB}\right) + S_K^2 - SC_K^2 = 0 \tag{8}$$

The basic equation for the stability analysis of sway-prevented frame on the elastic foundation with variable girder far ends conditions resulted from Eq.8 can be given as:

$$\frac{GAGB}{\beta_1\beta_2}\left(\pi/\overline{K}\right)^2 + \left(\frac{GA}{\beta_1} + \frac{GB}{\beta_2}\right)\left(1 - \frac{\pi/\overline{K}}{tan\left(\pi/\overline{K}\right)}\right) + \frac{2tan\left(\pi/2\overline{K}\right)}{\pi/\overline{K}} - 1 = 0 \tag{9}$$

Where $G = \dfrac{\sum\left(\frac{EI}{L}\right)_C}{\sum\left(\frac{EI}{L}\right)_g}$ (10)

The parameter β_n for braced frame can be given as:
For rigid girder far end:

$$\beta_n = \overline{S_n} - \overline{SC_n} = 2\lambda_n \frac{Cosh(\lambda_n) + Cos(\lambda_n)}{Sinh(\lambda_n) + Sin(\lambda_n)} \tag{11}$$

For fixed girder far end:

$$\beta_n = \overline{S_n} = 2\lambda_n \frac{Sinh(\lambda_n)\ Cosh(\lambda_n) - Sin(\lambda_n)\ Cos(\lambda_n)}{Sinh^2(\lambda_n) - Sin^2(\lambda_n)} \tag{12}$$

For hinged girder far end:

$$\beta_n = \frac{\overline{S_n^2} - \overline{SC_n^2}}{\overline{S_n}} = 2\lambda_n \frac{Cosh^2(\lambda_n) - Cos^2(\lambda_n)}{Sinh(\lambda_n)\ Cosh(\lambda_n) - Sin(\lambda_n)\ Cos(\lambda_n)} \tag{13}$$

2.2 Unbraced frame

By using the slop-deflection equations for the subassembly model for column in unbraced frame with girders on elastic foundation have variable girders far ends condition, the following relation can be obtained:

$$\begin{vmatrix} SC_K & \frac{\beta_2}{GB} + S_K & -\left(S_K + SC_K\right) \\ \frac{\beta_1}{GA} + S_K & SC_K & -\left(S_K + SC_K\right) \\ -\left(S_K + SC_K\right) & -\left(S_K + SC_K\right) & 2\left\{S_K + SC_K - \frac{1}{2}\left(\frac{\pi}{\overline{K}}\right)^2\right\} \end{vmatrix} = 0 \tag{14}$$

Eq.14 can be written in the following form:

$$\frac{3\left(\beta_1+\beta_2\right)}{GA\ GB}\left(\left(\frac{\pi}{\overline{K}}\right)^2 - 2S_K - 2SC_K\right) + \left(\frac{\beta_1}{GA} + \frac{\beta_2}{GB}\right)\left(\left(SC_K\right)^2 - \left(S_K\right)^2 + S_K\left(\frac{\pi}{\overline{K}}\right)^2\right) + \left(\frac{\pi}{\overline{K}}\right)^2\left(\left(S_K\right)^2 - \left(SC_K\right)^2\right) = 0 \tag{15}$$

The basic equation for stability analysis of unbraced frame on elastic foundation with variable girder far ends conditions resulted from Eq.15 can be given by:

$$\frac{GAGB\left(\pi/\overline{K}\right)^2 - \beta_1\beta_2}{\left(\frac{\beta_1+\beta_2}{2}\right)GAGB} - \frac{\pi/\overline{K}}{tan(\pi/\overline{K})} = 0 \tag{16}$$

The parameter β_n for unbraced frame can be given as:
For rigid girder far end

$$\beta_n = \overline{S_n} + \overline{SC_n} = 2\lambda_n \frac{Cosh(\lambda_n) - Cos(\lambda_n)}{Sinh(\lambda_n) - Sin(\lambda_n)} \tag{17}$$

For fixed girder far end

$$\beta_n = \overline{S_n} = 2\lambda_n \frac{Sinh(\lambda_n)\ Cosh(\lambda_n) - Sin(\lambda_n)\ Cos(\lambda_n)}{Sinh^2(\lambda_n) - Sin^2(\lambda_n)} \tag{18}$$

For hinged girder far end

$$\beta_n = \frac{\overline{S_n^2} - \overline{SC_n^2}}{\overline{S_n}} = 2\lambda_n \frac{Cosh^2(\lambda_n) - Cos^2(\lambda_n)}{Sinh(\lambda_n)\ Cosh(\lambda_n) - Sin(\lambda_n)\ Cos(\lambda_n)} \tag{19}$$

2.3 Simplified form of parameter βn

For the practical and design purposes, simplified formulae of parameter β_n were developed by curve-fitting as following:

2.3.1 When λn< 4

2.3.1.1 The case of column in braced frame

Rigid far ends

$$\beta_n = 2 + 0.32\lambda_n - 0.714\lambda_n^2 + 0.55\lambda_n^3 - 0.074\lambda_n^4 \tag{20}$$

Fixed far ends

$$\beta_n = 4 + 0.2\lambda_n - 0.42\lambda_n^2 + 0.31\lambda_n^3 - 0.039\lambda_n^4 \tag{21}$$

Hinged far ends

$$\beta_n = 3 - 0.104\lambda_n - 0.112\lambda_n^2 + 0.295\lambda_n^3 - 0.046\lambda_n^4 \tag{22}$$

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2.3.1.2 The case of column in unbraced frame

Rigid far ends

$$\beta_n = 6 + 0.0258\lambda_n - 0.066\lambda_n^2 + 0.04584\lambda_n^3 \quad (23)$$

Fixed far ends

$$\beta_n = 4 + 0.2\lambda_n - 0.42\lambda_n^2 + 0.31\lambda_n^3 - 0.039\lambda_n^4 \quad (24)$$

Hinged far ends

$$\beta_n = 3 - 0.104\lambda_n - 0.112\lambda_n^2 + 0.3\lambda_n^3 - 0.0474\lambda_n^4 \quad (25)$$

2.3.2 When λn ≥ 4

In this case, the close form solution (Eq.s 11, 12, 13, 17, 18 and 19) gives the following result:

$$\frac{Cosh(\lambda_n)+Cos(\lambda_n)}{Sinh(\lambda_n)+Sin(\lambda_n)} \approx \frac{Sinh(\lambda_n)\ Cosh(\lambda_n)-Sin(\lambda_n)\ Cos(\lambda_n)}{Sinh^2(\lambda_n)-Sin^2(\lambda_n)} \approx$$
$$\frac{Cosh(\lambda_n)-Cos(\lambda_n)}{Sinh(\lambda_n)+Sin(\lambda_n)} \approx \frac{Cosh^2(\lambda_n)-Cos^2(\lambda_n)}{Sinh(\lambda_n)\ Cosh(\lambda_n)-Sin(\lambda_n)\ Cos(\lambda_n)} \approx 1 \quad (26)$$

2.3.2.1 Case of column in braced or unbraced frame

The parameter β_n can be obtained by according to Eq. 26. Then, Eq.s 11, 12, 13, 17, 18 and 19 can be written as shown in Eq.27:

$$\beta_n \left\{ \begin{matrix} rigid\ \ far\ \ ends \\ fixed\ \ far\ \ ends \\ hinged\ \ far\ \ ends \end{matrix} \right\} = 2\lambda_n \quad (27)$$

From Eq.s 20 to 25, it can be concluded that, when the parameter λ_n approach to zero, Eq.s 20 to 25are reducing to the following form:

For braced column

$$\beta_n = \begin{Bmatrix} 2 & for\ \ rigid\ \ far\ \ end \\ 4 & for\ \ fixed\ \ far\ \ end \\ 3 & for\ \ hinged\ \ far\ \ end \end{Bmatrix} \quad (28)$$

For unbraced column

$$\beta_n = \begin{Bmatrix} 6 & for\ \ rigid\ \ far\ \ end \\ 4 & for\ \ fixed\ \ far\ \ end \\ 3 & for\ \ hinged\ \ far\ \ end \end{Bmatrix} \quad (29)$$

Eq.s 28 and 29 considered the effects of far end conditions of prismatic girders without interaction between frame and elastic foundation. In the case of sway-prevented column with rigid girders far ends, Eq. 28 given that $\beta_1 = \beta_2 = 2$, when substitute these beta values in Eq. 9 the resulted equation is as same as the US National codes equation (AISC, 2012, AISC, 1989, ACI, 2014, AASHTO, 1989).Also,Eq.29 given that $\beta_1 = \beta_2 = 6$ for sway-allowed frame with rigid girders far ends and when apply these beta values inEq.16, the resulted equation is as same as US National codes equation (AISC, 2012, AISC, 1989, ACI, 2014, AASHTO, 1989) for sway-allowed column.

3 Modified Alignment Chart (Exact Second Approach)

The modified $\overline{G}$-factor equation is proposed by Chen at el (1993a, 1993b) for the case of column in frame restrained by tapered girders using stiffness modification parameter. This paragraph introduces a modified $\overline{G}$-factor used in the current alignment charts of US National codes which enable the designer to calculate the modified K-factor by direct use of the alignment charts (Fig. 2).The $\overline{G}$ factor equation which considered the effect of elastic foundation and girders far end condition is given as:

$$\overline{G} = \frac{\sum E_c I_c / L_c}{\sum \gamma\ E_g I_g / L_g} \quad (30)$$

Where γ : is girder on elastic foundation stiffness modification parameter.

3.1 Exact form of stiffness modification parameter

The girder stiffness modification parameter γ is calculated for braced and unbraced frames by dividing the bending stiffness of girder on elastic foundation on the bending stiffness of ordinary girder not interacted with elastic foundation. The γ parameter is considered the effects of girder far end condition and elastic foundation stiffness.

For braced columns

When girder far end is rigid

$$\gamma = \frac{\overline{S} - \overline{SC}}{2} = \lambda_n \frac{Cosh(\lambda_n)+Cos(\lambda_n)}{Sinh(\lambda_n)+Sin(\lambda_n)} \quad (31)$$

When girder far end is fixed

$$\gamma = \frac{\overline{S}}{2} = \lambda_n \frac{Sinh(\lambda_n)\ Cosh(\lambda_n)-Sin(\lambda_n)\ Cos(\lambda_n)}{Sinh^2(\lambda_n)-Sin^2(\lambda_n)} \quad (32)$$

When girder far end is hinged

$$\gamma = \frac{\overline{S}^2 - \overline{SC}^2}{2\overline{S}} = \lambda_n \frac{Cosh^2(\lambda_n)-Cos^2(\lambda_n)}{Sinh(\lambda_n)\ Cosh(\lambda_n)-Sin(\lambda_n)\ Cos(\lambda_n)} \quad (33)$$

For unbraced columns

When girder far end is rigid

$$\gamma = \frac{\overline{S} + \overline{SC}}{6} = \frac{\lambda_n}{3} \frac{Cosh(\lambda_n)+Cos(\lambda_n)}{Sinh(\lambda_n)+Sin(\lambda_n)} \quad (34)$$

When girder far end is fixed

$$\gamma = \frac{\overline{S}}{6} = \frac{\lambda_n}{3} \frac{Sinh(\lambda_n)\ Cosh(\lambda_n)-Sin(\lambda_n)\ Cos(\lambda_n)}{Sinh^2(\lambda_n)-Sin^2(\lambda_n)} \quad (35)$$

When girder far end is hinged

$$\gamma = \frac{\overline{S}^2 - \overline{SC}^2}{6\overline{S}} = \frac{\lambda_n}{3} \frac{Cosh^2(\lambda_n)-Cos^2(\lambda_n)}{Sinh(\lambda_n)\ Cosh(\lambda_n)-Sin(\lambda_n)\ Cos(\lambda_n)} \quad (36)$$

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3.2 Simplified form of stiffness modification parameter

For the practical and design purposes, a simplified formulae of stiffness modification parameter γ can be obtained by compare Eq.s 11,12,13,17,18 and 19 with Eq.s 31 to 36.The following relation between parameter β and parameter γ can be concluded as follow

For braced frame $\gamma = \dfrac{\beta}{2}$ (37)

For unbraced frame $\gamma = \dfrac{\beta}{6}$ (38)

By using Eq.s 37and 38, the simplified form of stiffness modification parameter can be obtained as:

3.2.1 When λn< 4

3.2.1.1 The case of column in braced frame

Rigid far end $\gamma_n = 1 + 0.16\lambda_n - 0.357\lambda_n^2 + 0.275\lambda_n^3 - 0.037\lambda_n^4$ (39)

Fixed far end $\gamma_n = 2 + 0.1\lambda_n - 0.21\lambda_n^2 + 0.155\lambda_n^3 - 0.0194\lambda_n^4$ (40)

Hinged far end $\gamma_n = 1.5 - 0.052\lambda_n - 0.056\lambda_n^2 + 0.1475\lambda_n^3 - 0.023\lambda_n^4$ (41)

3.2.1.2 The case of column in unbraced frame

Rigid far end $\gamma_n = 1 + 0.0043\lambda_n - 0.011\lambda_n^2 + 0.00764\lambda_n^3$ (42)

Fixed far end $\gamma_n = 0.66 + 0.034\lambda_n - 0.071\lambda_n^2 + 0.052\lambda_n^3 - 0.0065\lambda_n^4$ (43)

Hinged far end $\gamma_n = 0.5 - 0.0173\lambda_n - 0.0187\lambda_n^2 + 0.05\lambda_n^3 - 0.0079\lambda_n^4$

(44)

3.2.2 When λn ≥ 4

3.2.2.1 The case of column in braced frame

The parameter γ_n can be obtained by substitute Eq. 37 in Eq.27

$$\gamma_n \begin{Bmatrix} rigid \quad far \quad end \\ fixed \quad far \quad end \\ hinged \quad far \quad end \end{Bmatrix} = \lambda_n \qquad (45)$$

3.2.2.2 The case of column in unbraced frame

The parameter γ_n can be obtained by substitute Eq. 38 in Eq.27

$$\gamma_n \begin{Bmatrix} rigid \quad far \quad end \\ fixed \quad far \quad end \\ hinged \quad far \quad end \end{Bmatrix} = \lambda_n / 3 \qquad (46)$$

When parameter λ_n approach to zero (the case of no interaction between girder and elastic foundation) Eq.s 39 to 44is reducing to the following form:

For braced column

$$\gamma_n = \begin{Bmatrix} 1 & for \quad rigid \quad far \quad end \\ 2 & for \quad fixed \quad far \quad end \\ 1.5 & for \quad hinged \quad far \quad end \end{Bmatrix} \qquad (47)$$

For unbraced column

$$\gamma_n = \begin{Bmatrix} 1 & for \quad rigid \quad far \quad end \\ 2/3 & for \quad fixed \quad far \quad end \\ 0.5 & for \quad hinged \quad far \quad end \end{Bmatrix} \qquad (48)$$

Eq.s 47 and 48aresame as the stiffness modification parameter proposed by Chen et al (1993b) for prismatic girders and SSRC-Guide(Johnston, 1979).Fig.3 shows a graphing of modification stiffness parameter γ_n for exact closed form solution (Eq.s 31 to 36) and simplified form solution (Eq.s 39 to 46).

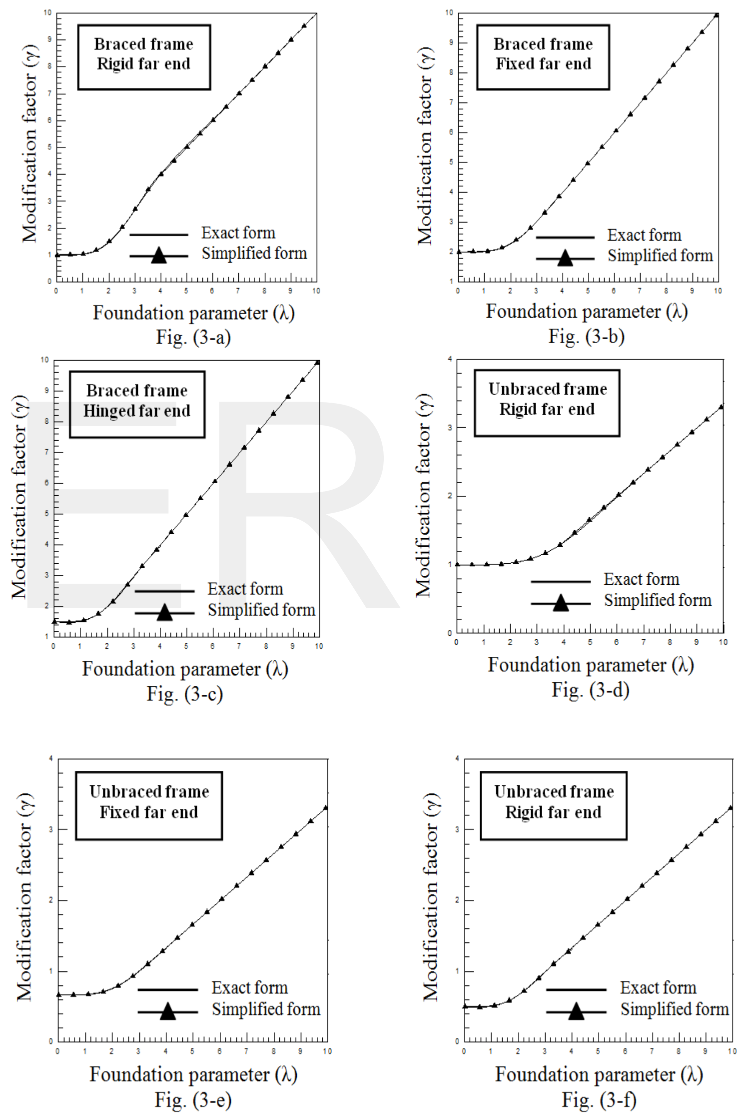

Fig.3. The relation between λ and γ

4 APPROXIMATE MODIFIED K-FACTOR PROPOSED FORMULAE

The following formulae involved modifying Newmark formula for braced frame and French “CM 66” formula for unbraced frame(Dumonteil, 1992),the modified approximate K-factor proposed equations in term of β_n can be given as:

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For braced column

$$\overline{K} = \sqrt{\frac{(G_A + 0.205\beta_1)\,(G_B + 0.205\beta_2)}{(G_A + 0.41\beta_1)\,(G_B + 0.41\beta_2)}} \tag{49}$$

For unbraced column

$$\overline{K} = \sqrt{\frac{\dfrac{57.6 G_A G_B}{\beta_1 \beta_2} + 24(\dfrac{G_A}{\beta_1} + \dfrac{G_B}{\beta_2}) + 7.5}{6(\dfrac{G_A}{\beta_1} + \dfrac{G_B}{\beta_2}) + 7.5}} \tag{50}$$

The exact Eq.s 9, 16 and approximate Eq.s 49 and 50 used when the two adjacent girders in one joint have same value of elastic foundation parameter λ. For the general case when the two adjacent girders in one joint have different value of elastic foundation parameter λ, the following formulae can be used:

For braced column

$$\overline{K} = \sqrt{\frac{(\overline{G_A} + 0.41)\,(\overline{G_B} + 0.41)}{(\overline{G_A} + 0.82)\,(\overline{G_B} + 0.82)}} \tag{51}$$

For unbraced column

$$\overline{K} = \sqrt{\frac{1.6\overline{G_A}\,\overline{G_B} + 4(\overline{G_A} + \overline{G_B}) + 7.5}{\overline{G_A} + \overline{G_B} + 7.5}} \tag{52}$$

Eq.s 51, and 52 can be considered as approximate solution of the following general case equations (Eq.s 53 and 54).

For braced column

$$\frac{\overline{GA}\;\overline{GB}}{4}\left(\pi / \overline{K}\right)^2 + \left(\frac{\overline{GA} + \overline{GB}}{2}\right)\left(1 - \frac{\pi / \overline{K}}{tan\left(\pi / \overline{K}\right)}\right) + \frac{2 tan\left(\pi / 2\overline{K}\right)}{\pi / \overline{K}} - 1 = 0 \tag{53}$$

For unbraced column

$$\frac{\overline{GA}\;\overline{GB}\left(\pi / \overline{K}\right)^2 - 36}{6\overline{GA}\;\overline{GB}} - \frac{\pi / \overline{K}}{tan(\pi / \overline{K})} = 0 \tag{54}$$

When the two adjacent girders in one joint have same value of elastic foundation parameter λ, Eq.s 49 and 50 can be written in term of γ_n as:

For braced column

$$\overline{K} = \sqrt{\frac{(G_A + 0.41\gamma_1)\,(G_B + 0.41\gamma_2)}{(G_A + 0.82\gamma_1)\,(G_B + 0.82\gamma_2)}} \tag{55}$$

For unbraced column

$$\overline{K} = \sqrt{\frac{\dfrac{1.6 G_A G_B}{\gamma_1 \gamma_2} + 4(\dfrac{G_A}{\gamma_1} + \dfrac{G_B}{\gamma_2}) + 7.5}{\dfrac{G_A}{\gamma_1} + \dfrac{G_B}{\gamma_2} + 7.5}} \tag{56}$$

Graphing of Eq.s 9 and 49 for the case of braced frame and Eq.s 16 and 50 for the case of unbraced frame with various girder far end condition are shown bellow.

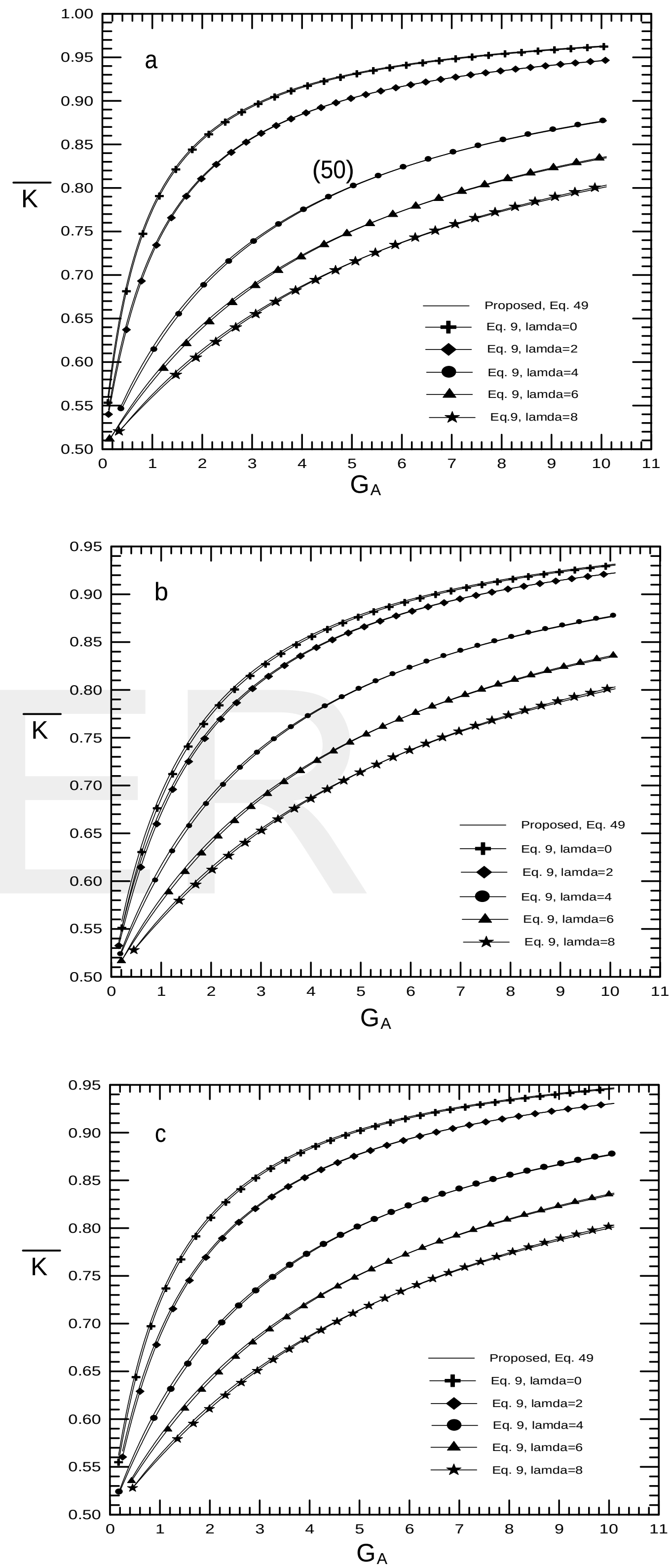

Fig. 4. The relation between modified effective length factor and GA for braced column, (a) for rigid girders far end, (b) for fixed girders far end, (c) for hinged girders far end, GA=GB, λ1=λ2=λ3=λ4=λ

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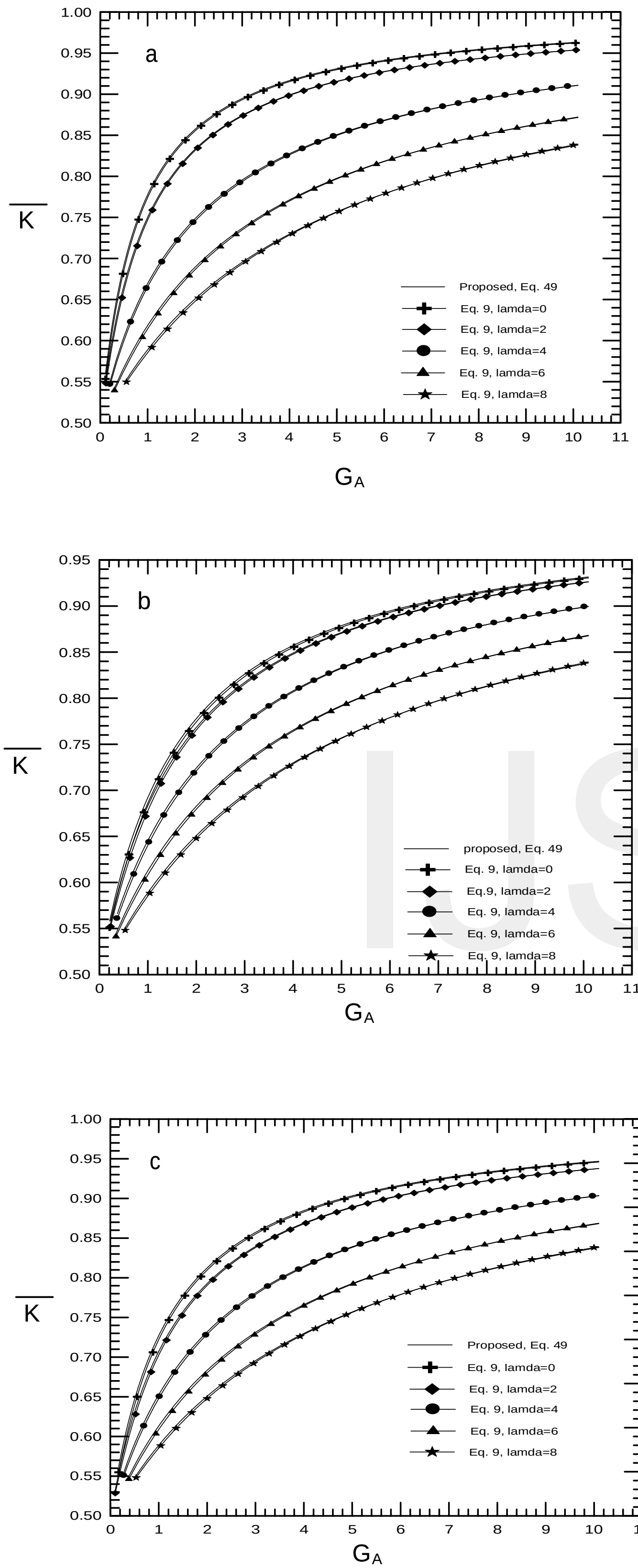

Fig.5. The relation between modified effective length factor and GA for braced column, (a) rigid girders far end, (b) fixed girders far end, (c) hinged girders far end, GA=GB, λ1=λ2=λ/2,λ3 = λ4= λ

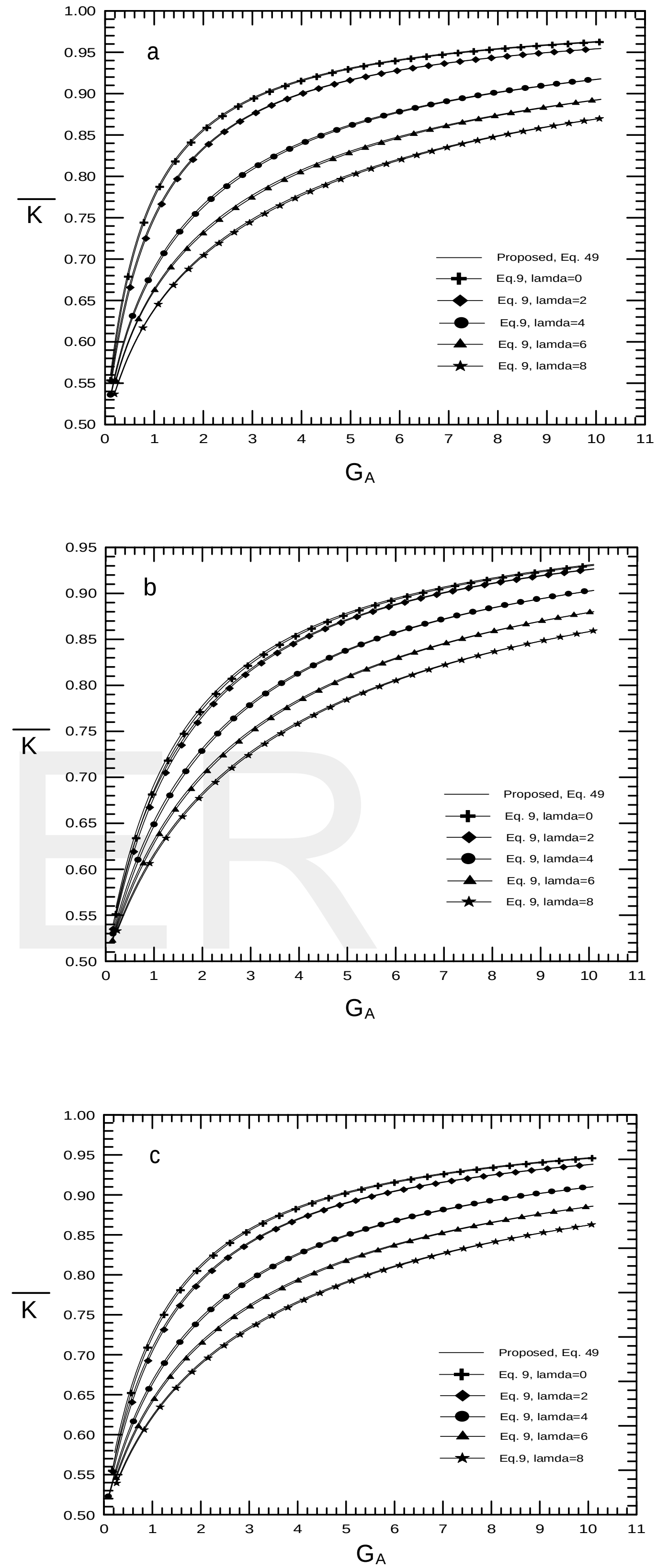

Fig. 6. The relation between modified effective length factor and GA for braced column, (a) for rigid girders far end, (b) for fixed girders far end, (c) for hinged girders far end, GA=GB, λ1=λ2=λ/4,λ3 = λ4= λ

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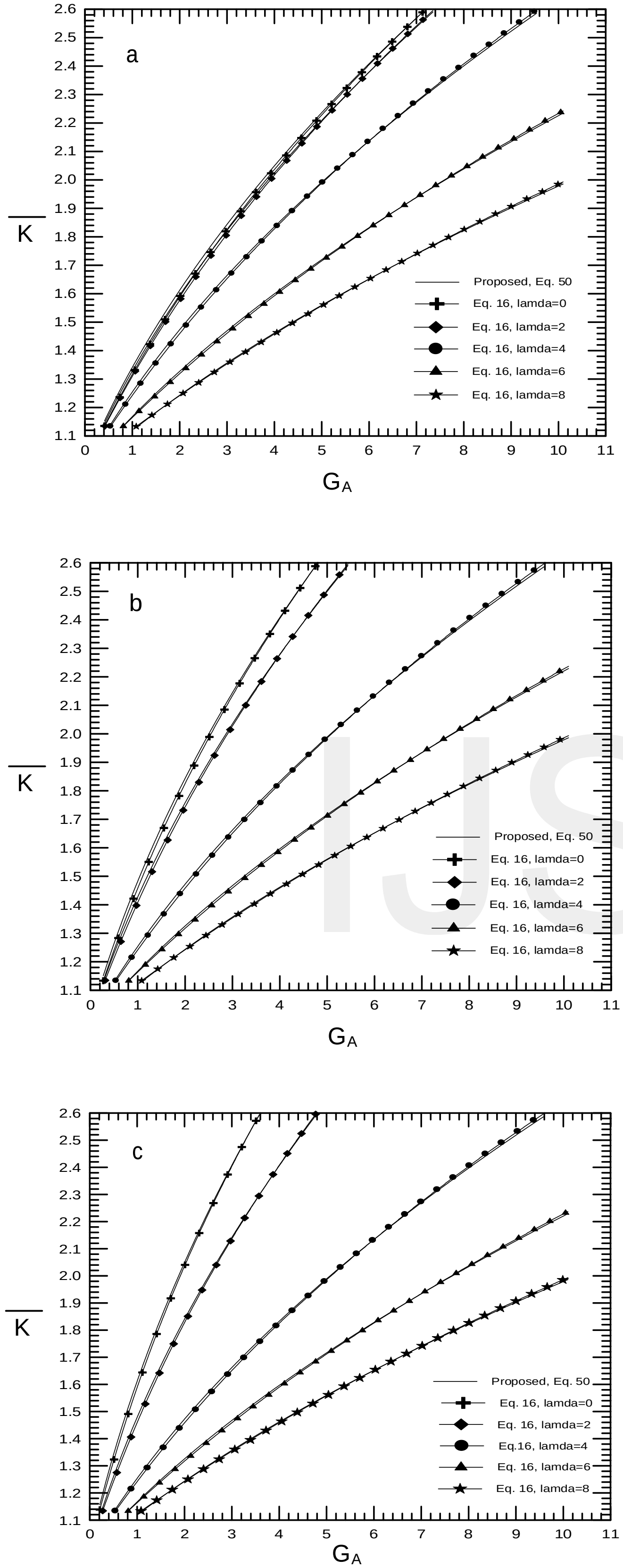

Fig. 7. The relation between modified effective length factor and GA for unbraced column, (a) for rigid girders far end, (b) for fixed girders far end, (c) for hinged girders far end, GA=GB, λ1=λ2=λ3 = λ4= λ

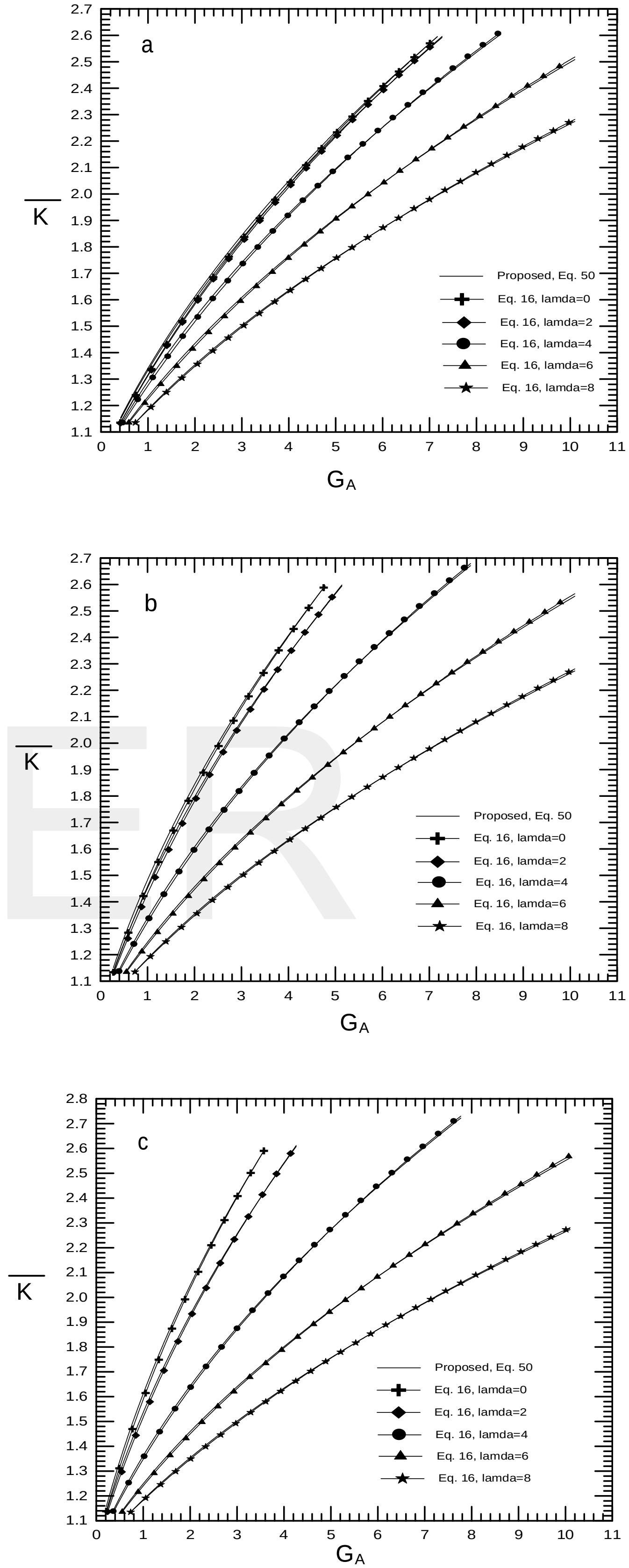

Fig. 8. The relation between modified effective length factor and GA for unbraced column, (a) for rigid girders far end, (b) for fixed girders far end, (c) for hinged girders far end, GA=GB, λ1=λ2=λ/2,λ3 = λ4= λ

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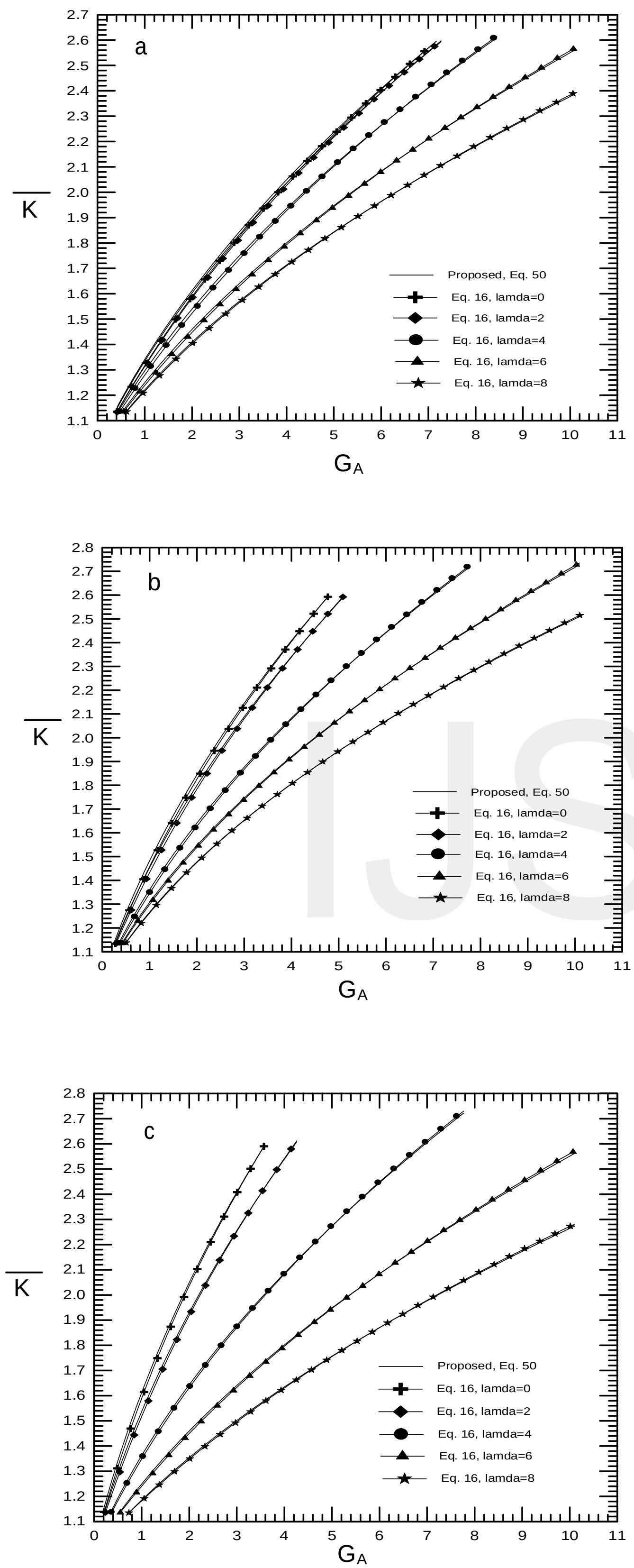

Fig. 9. The relation between modified effective length factor and GA for unbraced column, (a) for rigid girders far end, (b) for fixed girders far end, (c) for hinged girders far end, GA=GB, $\lambda 1=\lambda 2=\lambda/4$, $\lambda 3 = \lambda 4= \lambda$

5 Illustrative Examples

This section presents two examples, the first example is used to illustrate the calculation of modified K-factor for braced frame using exact equations and US codes alignment chart, and then check the accuracy and the validity of the modified K-factor proposed equations. The solution is obtained using the exact and approximate β, γ, and $\overline{G}$ factors. Some girders far end are modeled as fixed and hinged to prevent the side-sway. The second example is provided to illustrate the calculation of modified K-factor for unbraced frame. Exact equations and US codes alignment chart are adopted to check the accuracy of the approximate proposed equations, also the solution was carried out using β, γ, and $\overline{G}$ factors for the case of exact and approximate solution.

Example 1

A braced frame supported by girders on elastic foundation as shown in Fig. 10. The moment of inertia and element length are given in Table.1 below. The modulus of elasticity (E) was taken as a constant value for each element. The elastic foundation stiffness parameters are λ_{AB}=7, λ_{BC}= 7, λ_{HJ}=3 and λ_{QS}=2.5. Determine the effective length factor for columns DA, FB, HC and QH by using two approaches for exact and approximate solution.

Table 1
Length and moment of inertia for elements of example 1

Element	AB	BC	AD	BF	CH	DF	FH
I	5I	5I	3I	4I	2.5I	4I	4I
L	2L	L	1.5L	1.5L	1.5L	2L	L
Element	HJ	DM	FN	HQ	MN	NQ	QS
I	4I	2I	3I	2I	I	I	I
L	L	L	L	L	2L	L	L

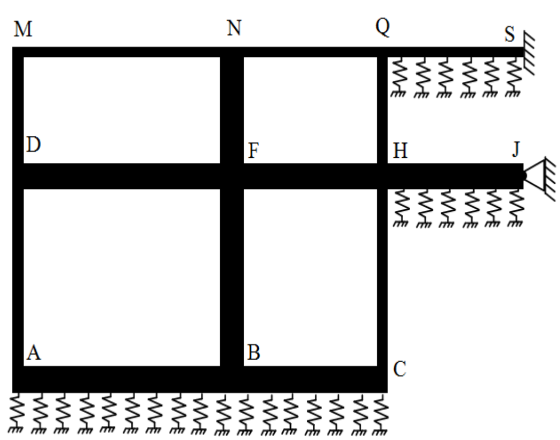

Fig.10. Frame of example 1

Solution : The calculation of exact and approximate values of parameter β using Eq.s 11 to 13 and Eq.s 20, 21, 22, and 27 respectively are illustrated in Table 2, Table 2also shows the exact values of parameter γ using Eq.s 31 to 33 and approximate γ values by depending on Eq.s39, 40, 41, and 45.

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TABLE 2
EXACT AND APPROXIMATE VALUES OF β AND γ OF EXAMPLE 1

λ	Girder far end	β Exact	β App.	γ Exact	γ App.
0	Rigid	2	2	1	1
7	Rigid	14	14	7	7
3	Hinged	-	-	2.982	2.96
2.5	Fixed	-	-	2.592	2.597

The values of G and $\overline{G}$ factors for each frame joint are tabulated in Table 3 using Eq. 10 and Eq. 30 respectively. Parameter γ illustrated in Table 2 used in calculations of factors $\overline{G}$ exact and $\overline{G}$ approximate.

TABLE 3
G AND $\overline{G}$ OF EXAMPLE 1

Joint	G	$\overline{G}$ Exact	$\overline{G}$ Approximate
A	0.8	0.1143	0.1143
D	2	2	2
B	0.356	0.0508	0.0508
F	0.945	0.945	0.945
C	0.34	0.0476	0.0476
H	0.458	0.23	0.231
Q	1	0.5568	0.556

For column DA, exact solution using Eq. 9

$$\Rightarrow \frac{1.6}{28}\left(\pi/\overline{K}\right)^2+\left(\frac{2}{2}+\frac{0.8}{14}\right)\left(1-\frac{\pi/\overline{K}}{\tan\left(\pi/\overline{K}\right)}\right)+\frac{2\tan\left(\pi/2\overline{K}\right)}{\pi/\overline{K}}-1=0$$

Using iteration exact $\overline{K}$=0.689 or by using Eq. 53 exact $\overline{K}$=0.689. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=0.689$.

Approximate solution using Eq. 49

$$\overline{K}=\sqrt{\frac{(2+0.205*2)\ (0.8+0.205*14)}{(2+0.41*2)\ (0.8+0.41*14)}}=0.692$$

Approximate solution using Eq. 51

$$\overline{K}=\sqrt{\frac{(2+0.41)\ (0.1143+0.41)}{(2+0.82)\ (0.1143+0.82)}}=0.692$$

Approximate solution by using Eq.55

$$\overline{K}=\sqrt{\frac{(2+0.41*1)\ (0.8+0.41*7)}{(2+0.82*1)\ (0.8+0.82*7)}}=0.692$$

For column FB, exact solution using Eq.9 or Eq.53 $\overline{K}$=0.637. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=0.636$. Approximate solution by using Eq. 49 or 51 or 55 $\overline{K}$=0.637.

For column HC, exact solution using Eq.53 $\overline{K}$=0.56. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=0.56$. Approximate solution using Eq. 51 $\overline{K}=0.567$.

For column QH, exact solution using Eq.53 $\overline{K}$=0.65. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=0.65$. Approximate solution using Eq. 51 $\overline{K}=0.654$.

TABLE 4
THE SUMMERY OF THE RESULTS OF EXAMPLE 1

Column	DA	FB	HC	QH
$\overline{K}$ exact equation	0.689	0.637	0.56	0.65
$\overline{K}$ alignment chart	0.689	0.636	0.56	0.65
$\overline{K}$ approximate	0.692	0.637	0.567	0.654

Example 2
For the unbraced frame shown in Fig.11, determine the effective length factor for columns HA, JB, MC, ND, QF, RH, VN and WP by using two approaches for exact and approximate solution. The elastic foundation stiffness parameters are λ_{AB}=6, λ_{BC}= 6, λ_{CD}=6, λ_{FH}= 2 and λ_{NP}=3. The values of moment of inertia and elements length are shown in Table 5 below and the modulus of elasticity (E) of each element is constant.

TABLE 5
LENGTH AND MOMENT OF INERTIA FOR ELEMENTS OF EXAMPLE 2

Element	I	L	Element	I	L
AB	3I	3L	DN	4I	5L
BC	3I	4L	FH	2I	2L
CD	3I	3L	HJ	4I	3L
AH	4I	5L	JM	4I	4L
BJ	6I	5L	MN	4I	3L
FQ	I	3L	QR	2I	2L
HR	3I	3L	RT	2I	3L
JT	4I	3L	TU	2I	4L
MU	4I	3L	UV	2I	3L
NV	4I	3L	VW	2I	2L
PW	I	3L			

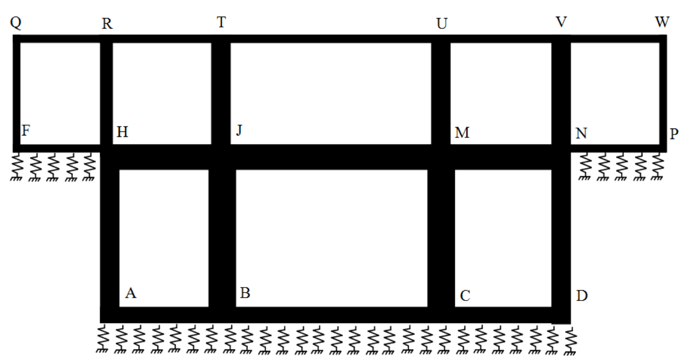

Fig.11. Frame of example 2

Solution: The calculation of exact and approximate values of parameter β according to Eq.s 17 to 19 and Eq.s 23, 24, 25, and 27 respectively are illustrated in Table 6, also the table shows the exact values of parameter γ using Eq.s 34 to 36and approximate γ values according toEq.s42, 43, 44, and 46.

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TABLE 6
EXACT AND APPROXIMATE VALUES OF β AND γ OF EXAMPLE 2

λ	Girder far end	β Exact	β App.	γ Exact	γ App.
0	Rigid	6	6	1	1
6	Rigid	11.92	12	1.99	2
3	Rigid	6.717	6.72	1.12	1.12
2	Rigid	6.15	6.154	1.025	1.025

The values of G and $\overline{G}$ factors for each frame joint are tabulated in Table 7 using Eq. 10 and Eq. 30 respectively. Parameter γ illustrated in Table 6 used in calculations of factors $\overline{G}$ exact and approximate.

TABLE 7
G AND $\overline{G}$ OF EXAMPLE 2

Joint	G	$\overline{G}$ Exact	$\overline{G}$ App.	Joint	G	$\overline{G}$ Exact	$\overline{G}$ App.
A	0.8	0.4	0.4	M	1.086	1.086	1.086
B	0.686	0.343	0.343	N	0.914	0.87	0.87
C	0.686	0.343	0.343	P	0.334	0.298	0.298
D	0.8	0.4	0.4	Q	0.334	0.334	0.334
F	0.334	0.326	0.326	R	0.6	0.6	0.6
H	0.77	0.763	0.763	V	0.8	0.8	0.8
J	1.086	1.086	1.086	W	0.334	0.334	0.334

For column HA, exact solution using Eq.54 $\overline{K}$=1.185. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.185$ approximate solution using Eq. 52 given as:

$$\overline{K}=\sqrt{\frac{1.6*0.763*0.4+4(0.763+0.4)+7.5}{0.763+0.4+7.5}}=1.207$$

For columns JB and MC, Exact solution using Eq. 16

$$\Rightarrow \frac{1.086*0.686\left(\pi/\overline{K}\right)^2-6*11.92}{\left(\frac{6+11.92}{2}\right)(1.086+0.686)}-\frac{\pi/\overline{K}}{tan(\pi/\overline{K})}=0$$

Using iteration exact $\overline{K}$= 1.2137 or by using Eq. 54 exact $\overline{K}$=1.2137. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.21$

Approximate solution using Eq. 50 given as:

$$\overline{K}=\sqrt{\frac{\frac{57.6*1.086*0.686}{6*11.92}+24(\frac{1.086}{6}+\frac{0.686}{11.92})+7.5}{6(\frac{1.086}{6}+\frac{0.686}{11.92})+7.5}}=1.244$$

Approximate solution using Eq.52 given as:

$$\overline{K}=\sqrt{\frac{1.6*1.068*0.343+4(1.068+0.343)+7.5}{1.068+0.343+7.5}}=1.244$$

Approximate solution using Eq. 56 given as:

$$\overline{K}=\sqrt{\frac{\frac{1.6*1.086*0.686}{1*1.99}+4(\frac{1.086}{1}+\frac{0.686}{1.99})+7.5}{\frac{1.086}{1}+\frac{0.686}{1.99}+7.5}}=1.244$$

For column ND, exact solution using Eq.54 $\overline{K}$=*1.2.* By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.2$ approximate solution using Eq. 52 $\overline{K}=1.224$

For column QF, exact solution using Eq. 16 given as:

$$\frac{0.334*0.334\left(\pi/\overline{K}\right)^2-6*6.15}{\left(\frac{6+6.15}{2}\right)(0.334+0.334)}-\frac{\pi/\overline{K}}{tan(\pi/\overline{K})}=0$$

Using iteration exact $\overline{K}$= 1.1091 or by using Eq. 54 exact $\overline{K}$=1.1091. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.1$, approximate solution using Eq. 50 or 52 or 56 $\overline{K}$=1.124

For column RH, exact solution using Eq.54 $\overline{K}$=1.215. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.21$, approximate solution using Eq. 52 $\overline{K}$=1.242

For column VN, exact solution usingEq.54 $\overline{K}$=1.272. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.27$ and approximate solution using Eq. 52 $\overline{K}$=1.292

For column WP, exact solution using Eq. 16

$$\frac{0.334*0.334\left(\pi/\overline{K}\right)^2-6*6.717}{\left(\frac{6+6.717}{2}\right)(0.334+0.334)}-\frac{\pi/\overline{K}}{tan(\pi/\overline{K})}=0$$

Using iteration exact $\overline{K}$= 1.1046 or by using Eq. 54, exact $\overline{K}$=1.1046. By using alignment chart with direct use of $\overline{G}$, $\overline{K}=1.105$

Approximate solution using Eq. 50

$$\overline{K}=\sqrt{\frac{\frac{57.6*0.334*0.334}{6*6.717}+24(\frac{0.334}{6}+\frac{0.334}{6.717})+7.5}{6(\frac{0.334}{6}+\frac{0.334}{6.717})+7.5}}=1.119$$

Approximate solution using Eq. 50 or 52 or 56 $\overline{K}$=1.119

TABLE 8
THE SUMMERY OF THE RESULTS OF EXAMPLE 2

Column	HA	JB	MC	ND
$\overline{K}$ exact equation	1.185	1.2137	1.2137	1.2
$\overline{K}$ alignment chart	1.185	1.244	1.244	1.2
$\overline{K}$ approximate	1.207	1.21	1.21	1.224
Column	QF	RH	VN	WP
$\overline{K}$ exact equation	1.1091	1.215	1.272	1.1046
$\overline{K}$ alignment chart	1.1	1.21	1.27	1.105
$\overline{K}$ approximate	1.124	1.242	1.292	1.119

6 THE SUMMARY AND CONCLUSIONS

This paper considered the determination of the effective length factor for column in braced and unbraced frames with girders on elastic foundation. The girders far ends were modeled as rigid, fixed or hinged. The exact formulae of the modified K-factor have been derived using two approaches; in the first solution technique of the modified effective length factor calculations were depended on parameter β. The Exact closed form and the simplified approximate form of parameter β have been derived. In the second solution approach, the calcu-

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lation described using girder stiffness modification parameter γ which provided the ability of direct use of U.S. National codes alignment charts. Exact form and simplified form of parameter γ have been investigated. For practical use, some approximate formulae of modified K-factor with high accuracy are proposed. Two examples were solved to illustrate the calculation method and solution accuracy.

Based on the results that obtained in the current study, several conclusions can be drawn. These conclusions are summarized as follows:

1- The two solution approaches of calculating the modified K-factor using parameter β or γ gives same modified K-factor results.
2- Fig. 3 shows an excellent agreement between the closed form of parameter γ (Eq.s 31, 32, and 33) and simplified form (Eq.s 39, 40, 41, and 45) for braced frame, also, for unbraced frame the figure indicated an excellent agreement between closed form (Eq.s 34,35, and 36) and simplified form (Eq.s 42, 44, 43 and 46). Thus, simplified form of parameter γ is practical for design purposes.
3- In the second solution approach, the US National codes alignment charts gives simple exact solution of Eq. 53 and Eq. 54 without need iteration.
4- Fig.s 4, 5, and 6 shows excellent agreement between the modified K-factor proposed equation (Eq. 49) and exact solution using Eq. 9 for the case of braced frame. Also, for the case of unbraced frame, Fig.s 7, 8, and 9 show excellent agreement between the exact solution using Eq. 16 and approximate proposed solution using Eq. 50. The percentage decrease in the modified K-factor for both braced and unbraced frame become more significant as the elastic foundation stiffness parameter (λ) increases.
5- Results summary of example 1 and example 2 showed that the percent of error in modified K-factor values between exact and approximate solution for braced frame is less than 1.25 %, while for unbraced frame is less than 2.23 %.

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